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| **National University of Computer and Emerging Sciences, Lahore Campus** | | | | |
| C:\Users\saif\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.Word\final design.jpg | **Course:** | **Database Systems** | **Course Code:** | **CS219** |
| **Program:** | **BS(Computer Science)** |  |  |
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| **Practice Problem:** | **FDs & NFS - SOLUTION** |  |  |

1. **[Find FDs]**

List all FDs.

|  |  |  |
| --- | --- | --- |
| **R** | | |
| **X** | **Y** | **Z** |
| x1 | y1 | z1 |
| x1 | y1 | z2 |
| x2 | y1 | z1 |
| x2 | y1 | z3 |

**ANSWER: *X →Y; Z →Y.***

1. **[Verify FDs]**

Which of the following FDs may or may not hold over schema S?

***a)*** *A → B*, **b)***BC → A*, **c)***B → C,* ***d*)***BC → D,* **e)***CD → B*

|  |  |  |  |
| --- | --- | --- | --- |
| **S** | | |  |
| **A** | **B** | **C** | **D** |
| 1 | 2 | 3 | 4 |
| 4 | 2 | 3 | 4 |
| 5 | 3 | 3 | 4 |

**ANSWER: FDs: (a), (c), (d) HOLD and FDs: (b), (e) NOT HOLD.**

1. **[Verify FDs]**

Which of the following FDs may or may not hold over schema R. Give valid reason.

**a)** A → CD, **b)** B → C, **c)** D → E, **d)** CD → E, **e)** E → CA

**R**

**A B C D E *Tuple#***

A1 B1 C1 D1 E1 *1*

A1 B2 C1 D1 E1 *2*

A2 B2 C1 D2 E3 *3*

A2 B3 C3 D2 E2 *4*

**ANSWER: a. Not Hold b. Hold c. Not Hold d. Hold e. Hold**

1. **[Prove Inference rules for FDs]**

Prove or disprove the following inference rules for functional dependencies. A proof can be made either by a proof argument or by using inference rules IR1 through IR6. A disproof should be done by demonstrating a relation instance that satisfies the conditions and functional dependencies in the left hand side of the inference rule but do not satisfy the conditions or dependencies in the right hand side.

a) {W →Y, X →Z} |= {WX →Y}

b) {X →Y} and Z subset-of Y |= {X →Z}

c) {X →Y, X →W, WY →Z} |= {X →Z}

d) {XY →Z, Y →W} |= {XW →Z}

e) {X →Z, Y →Z} |= {X →Y}

f) {X →Y, XY →Z} |= {X →Z}

**ANSWER:**

**a) Proof:**

**(1) W →Y (given)**

**(2) X →Z (given)**

**(3) WX →YZ (using IR5 (union) on (1) and (2))**

**(4) WX →Y (using IR4 (decomposition) on (3))**

**b) Proof:**

**(1) X →Y (given)**

**(2) Y →Z (using IR1 (reflexivity), given that Z subset-of Y)**

**(3) X →Z (using IR3 (transitivity) on (1) and (2))**

**c) Proof:**

**(1) X →Y (given)**

**(2) X →W (given)**

**(3) WY →Z (given)**

**(4) X →WY (using IR5 (union) on (1) and (2))**

**(5) X →Z (using IR3 (transitivity) on (4) and (3))**

**d) Disproof: X Y Z W**

**t 1 = x1 y1 z1 w1**

**t 2 = x1 y2 z2 w1**

**The above two tuples satisfy XY ->Z and Y ->W but do not satisfy XW ->Z**

**e) Disproof: X Y Z**

**t 1 = x1 y1 z1**

**t 2 = x1 y2 z1**

**The above two tuples satisfy X ->Z and Y ->Z but do not satisfy X ->Y**

**f) Proof:**

**(1) X →Y (given)**

**(2) XY →Z (given)**

**(3) X →XY (using IR2 (augmentation) to augment (1) with X)**

**(4) X →Z (using IR3 (transitivity) on (3) and (2))**

1. **[Closure]**

Consider the following relation and compute the closure of {A}+, {B}+, {C}+, {D}+, and {CD}+. Show your work.

|  |  |  |  |
| --- | --- | --- | --- |
| **R** | | |  |
| **A** | **B** | **C** | **D** |
| 1 | 2 | 3 | 4 |
| 4 | 2 | 3 | 4 |
| 5 | 3 | 3 | 4 |

**ANSWER:**

A+={ABCD}, B+={BCD}, C+={CD}, D+={CD}, and CD+={CD}.

1. **[Closure+Key]**

Consider the relation R (A, B, C, D, E, F) and the set F = {A****B, C****DF, AC****E, D****F}.

Find the closure of A and C (i.e. A+ and C+). What is the KEY of this relation? Prove it.

**ANSWER: A+= {A, B}; C+= {C, D, F}; Key= {AC}.**

1. **[Closure+Key]**

Consider the relation R (A, B, C, D, E, F, G, H, K) and the set F = {A****BC, CD****, CG****E, H****G, B****D, F****G}.

Find the closure of A and BC (i.e. A+ and {BC}+). What is the KEY of this relation? Prove it.

**ANSWER: A+= {A, B, C, D, E, G, H}; {BC}+= {A, B, C, D, E, G, H}; Key1= {AFK} and Key2= {BCFK}.**

1. **[Key]**

Consider the relation SALES (transno, itemno, price, qty, seller, sregion)

and the set F = {{transno, itemno} **** qty, itemno **** price, transno **** seller, seller **** sregion}.

What is the KEY of this relation? Prove it.

**Ans: {transno, itemno}**

1. **[Key]**

Consider the relation R (A, B, C) and the set F = {A****C, C****A}.

What is the KEY of this relation? Prove it.

**ANSWER: {AB} & {BC} are keys.**

1. **[Key]**

Given relation R(A,B,C,D,E) with dependencies AB **** C, CD **** E, DE **** B

Is AB a candidate key of this relation?

If not, is ABD? Explain your answer.

**Ans: No, AB+ = {A,B,C}, a proper subset of {A,B,C,D,E} i.e. R**

**Yes, ABD+ = {A,B,C,D,E}**

1. **[Minimal Cover]**

Find the minimal cover for the following set of FDs for a relation R (A, B, C, D):

F = {A **** BC, B **** C, A **** B, AB **** C, AC **** D}

**Ans:**

**Fc** = {A **** B~~C~~, B **** C, ~~A~~ **~~~~** ~~B~~, ~~AB~~ **~~~~** ~~C~~, A~~C~~ **** D} OR

**Fc = {A** → **B, B** → **C, A** → **D}**

1. **[Minimal Cover]**

Find the minimal cover for the following set of FDs for a relation R (A, B, C, D, E, F):

F = {A **** BC, E **** C, D **** AEF, ABF **** BD}

**Ans: Fc = {A** → **B, A** → **C, E** → **C, D** → **A, D** → **E, D** → **F, A~~B~~F** → **~~B~~D}**

OR

**Fc = {A** → **BC, E** → **C, D** → **AEF, AF** → **D}**

1. **[Minimal Cover]**

Find the minimal cover for the following set of FDs for a relation R (A, B, C, D):

F = {C **** BD, BC **** AD}

**Ans:**

**Fc =** {C **** ABD} OR

**Fc =** {C **** B, C **** D,  ~~B~~C **** A, ~~BC~~ **~~~~** ~~D~~}

1. **[Minimal Cover]**

Find the minimal cover for the following set of FDs for a relation R (A, B, C, D, E, G, H):

F = {AB **** C, DEG **** H, A **** C, DE **** G}

**Ans: Key={ABDE}**

**Fc =** {~~AB~~ **~~~~** ~~C~~, DE~~G~~ **** H, A **** C, DE **** G} OR

**Fc =** {DE **** GH, A **** C}

1. **[Minimal Cover]**

Consider the relation schema *R(A, B, C, D),* with FDs *F = {AB →CD, C →A, AD→C, CD →AB, D →B}*. Find a minimal cover of *F* (i.e. Fc).

**ANSWER:**

**Fc = *{AB →CD, C →A, ~~AD→C~~, ~~CD →AB~~, D →B}***

**i.e. Fc = *{AB →CD, C →A, D →B}***

OR

**Fc = *{AB →~~C~~D, C →A, AD→C, ~~CD →AB~~, D →B}***

**i.e. Fc = *{AB →D, C →A, AD→C, D →B}***

1. **[Minimal Cover]**

Consider the relation schema *R(A B C D E F G H)* with FDs *F = { A →BCD, AD →E, EFG→H, F →GH }*. Find a minimal cover of *F* (i.e. Fc).

**ANSWER:**

***Fc = {A →BCD, A~~D~~ →E, ~~EFG→H~~, F →GH }***

***i.e. Fc = {A→BCDE, F→GH}***

1. **[Minimal Cover]**

Find two different minimal cover of *F= {A → BC, B → AC, C → AB}*. Show your work. Also find all possible keys of R.

**Ans:**

***Fc1 = {A → B, B → C, C → A}***

***Fc2 = {A → C, C → B, B → A}***

***Fc3 = {A → C, B → C, C → AB}***

***Fc4 = {A → B, B → AC, C → B}***

***Fc5 = {A → BC, B → A, C → A}***

***Keys are {A}, {B}, and {C}.***

1. **[Minimal Cover]**

Consider the relation schema *R (A, B, C, D, E, F),* with a set of FDs *F = {*A→ BC , FC→ D , D→ B , AB→ F , F→ C, AD→ E*}.* Compute the minimal cover for *F* (i.e. *Fc*). Show your work! Also find all possible keys of R.

**Ans:**

***Fc* = *{* ~~A→ BC~~ , F~~C~~→ D , D→ B , AB→ F , F→ C, A~~D~~→ E *}***

*or*

***Fc* = { A→ EF, F→ CD, D→ B }**

***Key is {A}.***

1. **[Equivalent Sets]**

Consider the following two sets of FDs. Check whether or not they are equivalent. Provide proper reason.

*F1 = {A→B, B→C, C→A}* and *F2 = {A→C, C→B, B→A}.*

**ANSWER:** *They are equivalent.*

1. **[Equivalent Sets]**

Consider the following two sets of FDs. Check whether or not they are equivalent. Provide proper reason.

*F1 = {A→C, B→C, C→AB}* and *F2 = {A→BC, B→A, C→A}.*

**ANSWER:** *They are equivalent.*

1. **[Equivalent Sets]**

Consider the following two sets of FDs:

F = {A*→*C, AC*→*D, E*→*AD, E*→*H} and G = {A*→*CD, E*→*AH}. Check whether they are equivalent.

**Ans: Yes**

**Proof:**

**IN G:**

1. A→CD (given)
2. **A**→**C (IR4 on 1)**
3. A→D (IR4 on 1)
4. **AC**→**D (IR2 to augment 3 with C on LHS)**
5. E→AH (given)
6. **E**→**H (IR4 on 5)**
7. E→A (IR4 on 5)
8. E→D (IR3 on 7 and 3)
9. **E**→**AD (IR5 on 7 and 8)**

**Hence G COVERS F.**

**IN F:**

1. A→C (given)
2. AC→D (given)
3. AA→D (IR6 on 2, replace A with C)
4. A→D (simplification of 3)
5. **A**→**CD (IR5 on 1 and 4)**
6. E→AD (given)
7. E→H (given)
8. E→A (IR4 on 6)
9. **E**→**AH (IR5 on 7 and 8)**

**Hence F COVERS G.**

**So F and G are equivalent.**

**OR with closure method:**

**Answer:**

To show equivalence, we prove that G is covered by F and F is covered by G.

Proof that G is covered by F:

{A} + = {A, C, D} (with respect to F), which covers A ->CD in G

{E} + = {E, A, D, H, C} (with respect to F), which covers E ->AH in G

Proof that F is covered by G:

{A} + = {A, C, D} (with respect to G), which covers A ->C in F

{A, C} + = {A, C, D} (with respect to G), which covers AC ->D in F

{E} + = {E, A, H, C, D} (with respect to G), which covers E ->AD and E ->H in F